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INTERNATIONAL SERIES OF MONOGRAPHS ON AERONAUTICS AND ASTRONAUTICS

THEORY OF ELASTIC THIN SHELLS

by

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Translation from the Russian edited by

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Columbia University

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TRANSLATION EDITOR'S PREFACE TO ENGLISH EDITION

The importance of shell theory in structural analysis is strikingly reflected in a surprising abundance of individual contributions to the subject, scattered in a large number of periodicals of many countries. Indeed, C.B. Biezeno of the Technical University of Delft, in his introduction to a recent International Symposium on the Theory of Thin Elastic Shells, estimated the annual number of papers on the theory of shells to exceed one hundred, this productivity being on the increase at a considerable rate.

Yet it is astonishing that the number of comprehensive treatments of shell theory in the form of textbooks or monographs remains rather small. What was particularly lacking in the English-speaking scientific community was a monograph which would present with sufficient rigor the mathematical foundations of shell theory and discuss in detail the approximate methods of solution. A monograph of precisely this kind was, however, available to Russian scientists, namely Dr. A.L. Gol'denveizer's "Theory of Elastic Thin Shells". Although it was published in 1953, it is to this day the only book which formulates as completely as possible the different sets of basic equations and various approximate methods of shell analysis emphasizing asymptotic integration.

In view of the unique character of the book it seemed advisable to make the material accessible to English-speaking scientists. The American Society of Mechanical Engineers (ASME) applied for a grant to the National Science Foundation to provide for the translation of this volume. The publication is undertaken jointly by the ASME and Pergamon Press.

The English edition differs somewhat from a straight translation of the Russian of 1953. Firstly, the author provided the present edition with a supplement which embodies the most recent advances of the method of asymptotic integration. Secondly, the book was brought up to date by including a considerable number of additions and recent references, which are spread throughout the text. A more detailed description of these changes is given in the author's own preface to the English edition. The scope and the level of the monograph as a whole are described fully in the author's preface to the Russian edition.

Owing to the fact that the author's amendments were received at a time when preparation of the text for press was far advanced, it was unfortunately impossible to incorporate all of them at appropriate places. The particular amendments which could not be integrated into the text have been collected into a separate section at the end, where they are detailed in numbered series; in addition, as they arise in the main text, the attention of the reader is called to them by the appropriate serial number suffixed as a superscript to a convenient word at the place concerned. This seemed to be the arrangement entailing least inconvenience for the reader under the given circumstances.

Participating in the translation of this book were: Professor M.P. Bieniek, Mr. Ralph E. Ekstrom, Dr. I. Malkin, Professor W.A. Nash and Professor H.P. Thielman, whose competent co-operation in this undertaking is gratefully acknowledged.

G. HERRMANN

Translation Editor

AUTHOR'S PREFACE TO ENGLISH EDITION

During the period of time which separates the publication of the book in Russian and in English, new results have appeared which have a direct bearing on the material of the book, and the author has attempted to take account of them by making occasional changes in the text. Further, some misprints and inaccuracies in the formulation have been corrected. Supplementary references have been included, chiefly to work which was published after the book had appeared in Russian.

While work on the book was in progress the author made an attempt to develop a method of asymptotic integration of partial differential equations which apparently had certain elements of novelty. It is this method which forms the basic of the presentation.

Soon after publication of the book (but without connection with it), an increased interest in the analysis of asymptotic phenomena in mathematical physics became apparent. A series of mathematical publications appeared which were devoted to asymptotic integration of partial differential equations, and it thus became necessary to relate these methods to those used in the book. For this reason the English edition includes an addendum which represents an extract from the author's article in the journal Uspekhi matematicheskikh nauk (Progress in Mathematical Sciences), Vol. 15, No. 5, page 95 (1960). In this article the basic ideas of the method used in the book are formulated, and its place among other asymptotic methods indicated. Further the addendum contains a discussion of problems not touched upon in the book which are related to the imposition of boundary conditions and to the influence of edge constraints on the state of stress in the shell.

As is indicated in the preface to the Russian edition, the book deals with a definite range of problems considered from a definite viewpoint. This left its imprint on the literature cited: only those publications are referred to here which are closest to the content of the book. They are not segregated in a separate listing in order not to give the impression that such a listing reflected the author's opinion on the most important accomplishments in the theory of shells in general.

P R E F A C E

The subject with which this book deals is theory of thin shells, constructed on the hypothesis of preservation of the normal element. It is assumed that the shell material is isotropic and obeys the generalized Hooke's law, and that the strains, displacements and angles of rotation are so small that second powers of these quantities may be neglected. The highly important but as yet insufficiently developed problem of plastic deformation of shells is not touched upon. Also outside the scope of the work are the non-linear problems of the theory of shells and the problem of the stability of equilibrium.

The theory of shells is one of the most important applied branches of the theory of elasticity. Thin shell-type constructions are finding application in the most diverse branches of technology such as, to name only some, aircraft construction, building of industrial premises and dwellings, chemical engineering, engine construction, and shipbuilding. This explains the increased interest in the theory of shells and the significant success achieved in this field in recent years, particularly in the Soviet Union. The results obtained in the field of shell analysis are presented in numerous scientific articles and are partly touched upon in the literature devoted to other questions. However, if we are to indicate books which appeared in the U.S.S.R. during the last decade and which are devoted entirely to the linear theory of shells, then we can mention only three monographs: V.Z. Vlasov's "General Theory of Shells", V.V. Novozhilov's "Theory of Thin Shells" and A.I. Lur'e's "Statics of Thin-Walled Shells". In these studies an extremely wide circle of ideas and practical results is reflected, but it would be an exaggeration to say that the problem is treated with exhaustive completeness. To such a task, of course, the present volume does not address itself either, but the author thinks it is reasonable to undertake an attempt to present a theory of shells from a somewhat more general point of view than was done in the monographs mentioned.

In the theory of shells, since it is a science of an evident applied character, one of the most important problems is the construction of an approximate method of analysis, and in particular the problem of removal, in the basic relations, of those quantities which cannot significantly influence the final results and which only introduce unwarranted difficulties into the analysis. To achieve this aim, two avenues of approach may be selected. One of them consists in a broad use of certain assumptions, justified by experiments, regarding the relative role of certain factors. The second consists in a mathematical analysis of the basic relations of the theory of shells. In this connection a distinction between the technical and the mathematical theory of shells began to be drawn in recent years. This terminology correctly reflects the basic features of the two directions indicated, but if we are speaking of the results, then the antithesis of "technical" and "mathematical" theory of shells loses its sense, because it turns out that all known approximate methods may be deduced with the aid of the mathematical analysis also, while the solutions obtained theoretically may easily be given a simple physical interpretation.

Both the technical and the mathematical theory of shells (if we are to continue to use these conventional terms) have their limits. Regarding the first of them, a fairly clear notion may be formed from Vlasov's book. The chief merit of the technical theory of shells is its relative simplicity and the familiarity of the practical engineer with those concepts with which it operates. However, this is at the same time also its drawback, because it is impossible in the framework of such simple notions to embrace cases that are at all complicated. The weakest point in the technical theory of shells is its formulation of the region of applicability of methods recommended by it, which is of a rather diffuse character and does not permit estimation of the possible errors.

The mathematical theory of elasticity is more difficult to master, but its horizon is wider, and this is important particularly in connection with the fact that the development of technology leads infallibly to more complicated models of analysis. Furthermore, as another advantage of the mathematical theory of elasticity, we may mention that it affords a possibility of defining more rigorously the region of applicability of approximate methods of analysis and gives estimates of their errors. Novozhilov and Lur'e rely essentially on a mathematical analysis of the equations of the theory of shells, but the possibilities inherent in such an approach are not fully displayed in their books, both authors having intentionally limited their presentation to a definite range of problems.

The author of the present volume saw his basic task in formulating, with all possible completeness, various approximate methods of analysis of shells, in establishing the range of applicability of each of these methods and in giving estimates, if only rough, of possible errors. The fundamental method of investigation selected was a mathematical analysis of the basic relations of the theory of shells, considered as a system of differential equations with a smaller parameter. This permits us to consider from a unified point of view all the approximate methods of analysis of shells, which are not at a first glance logically interconnected. The material presented in the book may be partially supplemented by the results of Vlasov, Novozhilov and Lur'e. There is a series of questions, important in applications, which the author for this reason thought it possible to investigate only in principle, since they were already studied in detail in the monographs of his predecessors. Such problems include: analysis of shells of revolution, analysis of shallow shells and certain methods of analysis of cylindrical shells of arbitrary shape.

The book is based to a considerable degree on the results of studies over many years by the author, and is arranged in the following manner.

In Part I are deduced the general formulation and equations of the theory of shells, which are based on the well-known hypothesis of the preservation of the normal element. The usual assumption that the middle surface of the shell is referred to the lines of curvature is discarded, and all relations of the general theory of shells are deduced in two versions:

a) in completely arbitrary curvilinear coordinates, and

b) in orthogonal, but in general not conjugate curvilinear coordinates: the methods of vector analysis, which are assumed to be sufficiently known to the reader, are extensively used here.

The first chapter of Part I contains a brief recital of certain notions from the theory of shells. They are required not only for an understanding of this part of the book but also for a series of questions connected with the construction of approximate methods of analysis of shells.

Part II is devoted to the membrane theory. This most widely used approximate method of analysis of shells was formulated at approximately the same time as the more general bending theory. It is often presented apart from any connection with the bending theory, as a theory of thin elastic bodies in which the stresses through the thickness are distributed almost uniformly. Such a point of view incontestably possesses the advantage of physical plausibility, but it does not provide a means of establishing what place is occupied by the membrane theory among other approximate methods of analysis, nor of establishing the range of applicability of the membrane theory. Precisely such questions are especially emphasized in this book: the membrane theory is treated here as a particular case of the bending theory and the membrane equations as a result of simplification of corresponding equations of the bending theory.

The literature relating to the analysis of shells by the membrane theory is extremely rich, and the author has not aimed at recording all the results obtained in this field. Apart from the general questions, which are concentrated in Chapter 5, only those problems are considered which are related to the analysis of shells of zero curvature (among them cylindrical and conical) and spherical shells, and it is shown in section 23 that the methods of analysis of spherical shells may be extended also to shells drawn on second-order surfaces of zero curvature. Such selection of the material is explained by the fact that, first, cylindrical, conical and spherical shells constitute the class of shells which is most widely encountered in real construction, and secondly, it is possible to obtain for such shells a general integral of membrane equations and most prominently illustrate the principal assumptions of the bending theory.

The questions relating to the membrane theory are touched upon also in later parts of the book, where the membrane theory is considered in its relation to other approximate methods of analysis of shells. For this reason the problem of establishing the ranges of applicability of the membrane theory is not raised in Part II. On the other hand, it seemed desirable to include such questions as the relation of the membrane theory to the theory of flexure of surfaces and to the theory of beam systems. This permits us to establish in succeeding parts how the geometric configuration of the shell influences the character of its state of stress, and to formulate certain assumptions using a terminology familiar to the engineer.

The mathematical apparatus used in Chapters 5 and 6 is elementary; to understand Chapters 7 through 9 a familiarity with the basic propositions of the theory of functions of a complex variable is necessary.

In Part III methods of analysis of circular cylindrical shells with the aid of trigonometric series are considered. This particular problem received special consideration for two reasons: first, circular cylindrical shells are encountered more frequently than others in actual structures, and secondly, the methods of analysis of circular cylindrical shells are relatively simple. This latter circumstance allows us to study in greater detail the character of their states of stress and to recognize those properties peculiar to shells of arbitrary shape which will be used later in constructing general approximate methods. By virtue of this, the circular cylindrical shell is like a kind of a sample (gauge) which may be used to verify the general approximate methods discussed in subsequent parts.

Major emphasis is given in Part III to the construction of approximate methods (for the most part methods known in the literature); these are deduced on the basis of a study of the exact relations and by discarding in them secondary terms. At the same time the basic idea of the whole book is gradually introduced, namely the need for specialization of approximate methods and their adaptation to problems of a definite type, such that the more complicated cases could be reduced to the ones already studied, by means of a suitable decomposition of the sought state of stress.

Part IV is essentially mathematical in character and its purpose is to justify the approximate methods of shell analysis. In this part of the book the equations of the theory of shells are considered as equations containing a small parameter h^* (relative half-thickness) at high derivatives, and the asymptotic properties (as $h^* \to 0$) of integrals of this system are studied. The basic apparatus of this investigation is the method of asymptotic integration. In application to equations with partial derivatives it encounters a series of obstacles, and the attempt to maintain complete mathematical rigor and to exhaust completely the questions raised therefore had to be renounced. The reader will note that in a series of cases the author was forced to limit the generality considerably, or to replace proofs by conjectures. Questions of convergence of asymptotic processes are not raised here at all.

In Part V approximate methods of analysis of shells are formulated which are based upon the results obtained earlier. The membrane theory, the approximate method most widely used in practical structural analysis, serves as a point of departure. Sufficient conditions of applicability of the membrane theory are established; following this, assumptions are successively introduced that this or that among these conditions is violated; and for each of these cases approximate methods are constructed which are based on the specific properties of the given problem. Such an approach, if it were possible to pursue it to the end, retaining the generality of the argument, would lead to a completely universal set of approximate methods of analysis of shells. However, it has not been possible to attain this end as yet. Certain practically important problems had to be left outside the scope of investigation, partly because too much space would be needed for their discussion, partly because it was not possible to study them sufficiently.

Estimates of asymptotic errors are given for the suggested approximate methods of analysis, that is, the law of decrease of errors as the relative thickness of the shells decreases without limits is established. The relative thicknesses of shells, as a rule, are rather small as compared to unity, and therefore asymptotic errors permit estimation of the errors in the usual sense of this term. However, deductions of this type should be made with a certain caution, particularly in those cases when the curvatures of the middle surface or the coefficients of its first quadratic form undergo significant changes in the region considered.

The presentation is arranged in such a manner that each part of the book represents a complete entity and may be read by itself. As a consequence, the author has not made an attempt to preserve a unity of style and has sometimes admitted minor repetitions.

The book touches on a defined range of problems, considering them from a defined point of view. This has left its imprint also on the choice of literature cited. Only those works are mentioned which are closest to the subject matter of our book. Nor are they grouped into a separate list, lest this should give the impression that such a list relfected the author's views on the most substantial achievements in shell theory in general.