MATHEMATICS CLASSROOMS: STUDENTS' ACTIVITIES AND TEACHERS' PRACTICES

Mathematics Classrooms: Students' Activities and Teachers' Practices

Edited by

Fabrice Vandebrouck
Université Paris Diderot, France

SENSE PUBLISHERS
ROTTERDAM / BOSTON / TAIPEI

A C.I.P. record for this book is available from the Library of Congress.

ISBN 978-94-6209-279-2 (paperback)
ISBN 978-94-6209-280-8 (hardback)
ISBN 978-94-6209-281-5 (e-book)

Published by: Sense Publishers,
P.O. Box 21858, 3001 AW Rotterdam, The Netherlands
https://www.sensepublishers.com/

Printed on acid-free paper

TABLE OF CONTENTS

BARBARA JAWORSKI

PREFACE

This book presents unique insights into a significant area of French research relating the learning and teaching of mathematics in school classrooms and their development. Having previously had only glimpses of this work, I have found the book fascinating in its breadth of theory, its links between epistemological, didactic and cognitive perspectives and its comprehensive treatment of student learning of mathematics, classroom activity, the work of teachers and prospective teacher development. Taking theoretical perspectives as their starting points, the authors of this volume present a rich array of theoretically embedded studies of mathematics teaching and learning in school classrooms.

The book charts the use of a theoretical/methodological perspective called *The Double Approach*, a didactic and ergonomic approach for the analyses of teaching practices (Robert & Rogalski, 2002). This approach is concerned simultaneously with the design of teaching and with its practical, ergonomic (work-based) contribution to students' learning of mathematics in classrooms. It seeks to address associated issues widely and in their full complexity recognising institutional dynamics and constraints, the impact of social and cultural perspectives and interweaving layers of activity.

The term "activity" is ubiquitous throughout, taking on two kinds of meaning, in one sense referring to the actions of students and teachers in the classroom and, in parallel, referring also to *activity* as in *Activity Theory*, a complex dynamic encompassing the wholeness of classroom learning and teaching; as Leont'ev has expressed it, "the non-additive, molar unit of life … a system with its own structure, its own internal transformations, and its own development" (Leont'ev, 1979, p. 46). The two senses are deeply entwined in the ways activity is addressed. Thus, it is not surprising that one of the foundations of the Double Approach, its "organising framework" is the sociohistorical theory of Vygotsky, and followers such as Leont'ev. In Chapter 1, Janine Rogalski writes, "the object of study consists of the activity of an individual subject with individual motivations, within a specific situation. When the subject is a teacher, it is not the "properties" or "functioning" of the teacher's position that is at issue. … Rather the issue involves questions of diversity among teachers, and the development and emergence of their individual professional competencies" (p. 3). The focus on the individual subject ("as a person-subject rather than as a didactic subject," ibid.) is perhaps somewhat more surprising, especially since it leads the authors to consider a Piagetian approach of epistemological genetics alongside Vygotsky's sociohistorical framework. The surprise is in the juxtapositioning of theories of Piaget and Vygotsky of which others scholars have been cautious, if not dismissive, due to the (supposed) incommensurability of these theoretical perspectives (see e.g., Lerman,

1996). More recently Lerman (2013) wrote "In general, drawing on more than one theoretical perspective needs some work in order to ensure that the perspectives are coherent together." Regarding the complementarities of the two areas of theory, Rogalski writes: "In particular, the Piagetian theory looks "from the student's side" at epistemological analyses of the mathematical objects in play, while the Vygotskian theory takes into account the didactic intervention of the teacher, mediating between knowledge and student in support of the students' activity" (p. 23). Just one of the exciting aspects of these authors' use of the Double Approach is to see how this theoretical juxtapositioning leads to analyses of teacher and classroom activity which make sense for the researchers and those who learn from this research.

So, what exactly is meant by the *Double Approach*? Aline Robert and Christophe Hache (Chapter 2) weave the abstract theory (above) with theoretical frameworks relating to teaching and learning in classrooms and general methodologies that follow from these frameworks. They write, "… we seek to measure the gap between the activities of students applying their knowledge (during its acquisition) analyzed *a priori*, and the activities that may actually have taken place during a regular lesson" (p. 62). At a simple level, we see an analytic progression from epistemological analysis in a mathematical topic, through a didactic analysis relating to the design of teaching, into analyses of classroom activity and inter-relations between teachers and students with, last but not least, analyses of student activity and understanding. The progression is not linear (as my list might suggest); the research lens may focus in any of these areas, or zoom out to address complex inter-relationships between them. Thus the programme is ambitious. The reader is taken through subdivisions of "the world of the study:" we read of student activity, the transition from designed task to student activity with task, levels of conceptualization (related to Vergnaud's, 1990, 'conceptual fields'), the nature of concepts and students' progression with concepts in terms of generalization and formalization, the knowledge of teachers and design of tasks, wider issues in terms of systemic demands or emotional, personal and social factors. For example, relationships between didacticians and teachers are addressed, ways in which teachers adopt or adapt didactic designs, the 'work' of the teacher in the classroom, teacher speech patterns and representations of mathematical concepts and their relationships to student activity. Consideration of the profession of teaching and roles in teachers' work lead to questions of teaching development and the education of new teachers.

In presenting theory and methodology in these areas, the authors move to and fro between the cognitive and the sociocultural frames so the reader is faced with challenges in making sense of the complexities involved. Unsurprisingly, this ambitious enterprise raises many questions for the reader, not least as to how theoretical complexities are translated into practice in schools and classrooms, how teachers work with researchers (or independently with these perspectives), and how researchers address inter-relationships between observations of student conceptualisation, teachers' didactic processing in design of activity and the wider frames of educational and sociocultural impact. For example, as the authors

acknowledge, "if it is difficult to analyse teaching in relation to learning, it is even more difficult to have legitimate evidence of it" (p. 58).

These questions are addressed variously in the (nine) chapters which follow in which we see the elements of the Double Approach in action with differing zooms of the research lens. Authors present a variety of methodological approaches with analyses of classroom settings, design of tasks, mathematical topics, teachers' intentions, student responses, imposed constraints and the degree of 'leeway' experienced by the teachers. For example, in Chapter 3, Eric Roditi discusses the tasks offered by four teachers in similar sixth grade classes on the topic of multiplication of decimal numbers. We read of the nature and choice of tasks, their levels of cognitive demand and their relation to curriculum guidance on the topic, suggested class time, and expectations of professional practice discerned through classroom observations and interviews with teachers. *A priori* expectations of students' activity according to designed tasks is compared with student outcomes in the tasks. Despite the commonalities of designed tasks in the four classrooms, research emphasised and categorized the variability of classroom activity depending on the ways in which individual teachers worked with students in their class.

The work of teachers and its relation with the realities of situation and context is central to methodologies employed. The relationship between teaching practices and student learning is a recurring theme. Julie Horoks writes in Chapter 6 (p. 135), "Naturally we are not questioning the teacher's work, and we will consider the different components of his/her job to explain certain choices made for his/her class." Horoks describes the use of classroom video recordings to reveal teachers' use of tasks focusing on similar triangles and to relate students' degrees of success with these tasks to the ways in which the tasks were used in the classroom. Monique Chappet-Paries, Aline Robert and Janine Rogalski, in Chapter 4, focus on classroom activity around the theorem of Pythagoras, analysing a teacher's speech patterns to gain insight into invariants in the teacher's practice and ways in which these invariants impact on students. The idea of a "teaching scenario" – a sequence of lessons and exercises around a mathematical topic such as decimal numbers, similar triangles or Pythagoras' theorem (studied a priori) – is a common theoretical construct. For example, in Chapter 7, Aurélie Chesnais discusses the implementation of the same teaching scenario (about orthogonal symmetry) by two experienced teachers in order to study regularities and variability of practices between teachers, as well as the relationship between teaching practices and student learning.

The major theme of relations between a priori analyses of tasks, teachers' implementation of tasks in the classroom and students' take-up of tasks is considered in later chapters with an added dimension, that of the use of electronic resources. In Chapters 8 and 9 these are *Electronic Exercise Bases*, consisting of mathematics exercises within an environment, which includes different types of suggestions, aids, tools (graphs, calculators, etc.), lesson reminders, as well as explanations, answer analyses or complete solutions. The scenarios here are designed around the electronic environment and its use by students with a study of,

for example, how the designed situations influence the students' activity. Research reveals that the expected activity is not always the activity developed by the students and emphasizes the difficulty for students to regulate their activity while interacting with the software without teacher intervention. As well, studies address the impact of the electronic resources on the day-to-day activities of teachers and on teachers' evolving classroom practices. Chapter 10 compares activity in a dynamic geometry environment with that in a pencil and paper environment to analyze how the tasks designed for ICT environments differ (or not) from those of non ICT ones. Research explored the differences in classroom management, including ways in which the teacher assisted students, in order to understand their possible impact on students' activities.

I have given these very brief sketches of the focus of various chapters to illustrate or exemplify the pervasive themes of the book in addressing classroom complexity and the deeply inter-related nature of teaching-learning activity. Each study presents different facets of design, implementation and impact of scenarios within the real constraints of classrooms and the personal and social influences which surround classroom interactions. In the final chapter, Maha Abboud-Blanchard and Aline Robert reflect on the earlier chapters to distil elements of their findings which offer insights that are useful in considering the education of mathematics teachers, and of those who will train mathematics teachers. They ask the question, "who should be trained first – the teacher or the teacher's trainer?" (p. 235). This leads to their setting out a training programme for the trainers of mathematics teachers. They acknowledge that this is speculative and that associated research in yet to be undertaken. It nevertheless points to the ambitious scope of the book and the broad programme of research it charts.

Throughout this book the reader is made aware of many unanswered questions and challenged to consider associated theoretical and methodological issues. There is nevertheless an internal consistency and coherence to this work which revolves around the Double Approach. For English-speaking communities who have lacked opportunity to access the French literature the book opens up a wealth of new ways of thinking about and addressing unresolved issues in mathematics learning, teaching and teacher education. I recommend it wholeheartedly!

References are included in the general bibliography of the book.

FABRICE VANDEBROUCK

INTRODUCTION

This book presents several works in the field of mathematics didactics revolving around secondary school and university teaching. The specificity of the research studies in question is that they attribute as much importance to the actors (students and teachers) as to the mathematics and the school situation. These studies fit well in the very general framework of Activity Theory.

The presented researches aspire to analyze what is at play in a mathematics classroom, by varying the school situations, the environments, the contents, the teachers and the classrooms. The main objective is to study, understand, and even interpret the links between the teaching of a given mathematical content and the corresponding student learning. We seek to highlight regularities and variations of these processes in order to better understand students' acquisitions, and interpret the teachers' practices. The work as a whole leads to inferences which can contribute to the professional development of teachers by widening the range of possible activities for each teacher.

The general framework of Activity Theory, with associated development theory, is described in chapter 1, and we directly clarify how this work fits in this framework. The analyses of students' in-class activities, as they are organized by the teachers, provide us with data which allow us to tackle teachers' practices and approach students' learning: the general theory accounts for this focus and the corresponding reality splitting. Nevertheless, the way activities are assigned to mathematics and school situations is not very present in the framework of Activity and development theories. Therefore, the necessary theoretical and methodological complements are presented in chapter 2.

The main concern of this book is however not theoretical, even though its specificity borrows elements from Activity Theory and development theories which complement typically didactical tools. We seek to assign to the singular subjects (students and teachers) their place within the didactical relationship, even though the affective and social factors are not directly accounted for, despite their high importance. We develop the means to collect and analyze in a significant way, adapted to our project, data about teaching and learning allowing us to interpret the relationship between the two.

All the research studies of this book follow a common methodology presented in chapter 2, but involve, of course, indispensable adaptations which are introduced gradually. They pertain to the teaching of mathematics in middle school, high school, or the first two years of university. Some works are the fruit of individual research[1] and handle a small number of cases in an exhaustive manner, often over quite short periods of time. Others works are clusters of research studies or the fruit

F. Vandebrouck (ed.), Mathematics Classrooms: Students' Activities and Teachers' Practices, 1–2.

of collective research based on larger data and oriented more directly towards results which are relevant for the general research question of the book. In any event, there is always a limit to the hasty generalizations of results. Hence, there are neither definitive results nor (even less) prescriptions in our discourse.

In chapters 1 and 2, we present the theoretical frameworks and the tools used in the book, while stating the specificity of our research. Chapters 3 and 4 are concerned with the results about teachers' practices in "ordinary" classrooms. They highlight the stability of teachers' practices and also account for the diversity and variability between the teachers. Chapter 5 deals with teaching manuals and shows that exercises proposed in these manuals do not offer the teachers opportunities to diversify their student activities. Chapters 6 and 7 refer more directly to teachers' practices in relation with students' activities. Chapter 8 focuses on the activity of students in a specific teaching situation in a computerized environment. Chapters 9 and 10 deal with teachers' practices in computerized environments, in particular the comparison of teachers' activities in different environments. Chapter 11 is a large scale study about teachers' practices and the factors related to the regularity and variability of the practices. Last, chapter 12 comes as a synthesis of the book with an opening on professional development of teachers.

The different chapters can be read in a relatively independent way. In particular, it is not necessary to complete an exhaustive reading of chapters 1 and 2 in order to read the other chapters … and vice versa!

NOTES

[1] All the researchers who contributed to this book, apart from Aurélie Chesnais, Eric Roditi and Janine Rogalski, are members of the Laboratoire de Didactique André Revuz (LDAR) at Paris Diderot University

Fabrice Vandebrouck
Laboratoire de Didactique André Revuz
Université Paris Diderot

JANINE ROGALSKI

1. THEORY OF ACTIVITY AND DEVELOPMENTAL FRAMEWORKS FOR AN ANALYSIS OF TEACHERS' PRACTICES AND STUDENTS' LEARNING

INTRODUCTION

The goal of this chapter is to propose a theoretic framework to analyze the structured activities of teachers and their students, and to provide support for some inferences regarding teachers' training in professional competencies[1] and students' acquisition of knowledge in specific disciplines.

The organizing framework is that of the theory of activity, which was established by Leontiev, enriched through a line of research originated by Vygotsky, and then exploited and developed within the field of ergonomic psychology (Leplat, 1997; Rogalski, 2004). Its fundamental components are:

- the distinction between task and activity;
- the double point of view, taking into account both the situation and the subject of the action; and
- the system of double regulation of activity, in which determining factors, and the effects of the activity, influence situational components as well as the subject. This regulation is not only retroactive, but also proactive, as a goal-oriented activity is affected when subjects adapts their actions in an attempt to produce the desired results.

Within this theoretic framework, the object of study consists of the activity of an individual subject, with individual motivations, within a specific situation. When the subject is a teacher, it is not the "properties" or "functioning" of the teacher's position that is at issue here (as would be the case for a *stricto senso* didactic perspective, which we could define as the "science of didactic processes"). Rather, the issue involves questions of diversity among teachers, and the development and emergence of their individual professional competencies. Equally relevant are considerations of the student as a person-subject, rather than a didactic subject. All this leads us to consider the Piagetian approach of epistemological genetics, together with Vygotsky's socio-historical framework, as they relate to individual development.

Taking into account the effects of the activity on the subject is an aspect of the developmental and constructivist dimension of the theory of activity (TA). Our focus is on the activity, on its determining factors and on its effects as they relate to teaching mathematics. We are particularly interested in the activity's effects on a teacher's development of professional competencies, and on a student's mathematical conceptualization. Interpreting TA within the theoretical frameworks

F. Vandebrouck (ed.), Mathematics Classrooms: Students' Activities and Teachers' Practices, 3–22.

of Piaget and Vygotsky enriches our approach, by defining the developmental dimension of the double regulation in terms of factors and effects, temporality, and the role of psychological tools (Vygotsky) and cognitive tools (Piaget). We include within the Vygotskian framework Bruner's findings on mediation (Wood, Bruner, & Ross, 1976), which add to our understanding of the didactic intervention of a teacher in class. We conclude with a discussion of these theoretical frameworks in order to define the tools we will use in our analysis of teaching practices and student activities in mathematics.

THE THEORY OF THE GOAL-ORIENTED ACTIVITY

The theory of activity was developed by researchers who followed Vygotsky in studying the psychology of work (later called "ergonomic psychology"). The theory was then used in professional didactics, before being "articulated" with a didactical approach to mathematics teaching, in the so-called "double approach" (Robert & Rogalski, 2002, 2005; Robert, chapter 2). The theory involves goal-oriented and motivated activities. By their actions, subjects aim to achieve task goals, and their actions are driven by motivations of the activity.

We will describe the following elements, all of which are essential to our objectives: the task-activity distinction, task structure, the various ways to analyze an activity, and the connection between the subject and the situation within the model of double regulation of activity. We will also indicate how this theoretical framework allows us to analyze the structure of teacher and student activities.

Task and activity

The task-activity distinction is central to the theory of activity. The activity relates to the subject,[2] while the task relates to the objects of the action. The definition of a *task*, as proposed by Leontiev (1975, 1984) and developed by Leplat (Leplat and Hoc, 1983; Leplat, 1997) is the "goal to be attained under certain circumstances." The *activity* is what a subject engages in during the completion of the task. This includes not only external actions, but also inferences, hypotheses, decisions, and actions the subject decide *not* to take. The activity also includes the subject's time management and personal state – workload, fatigue, stress, enjoyment of work – as well as interactions with others within the work situation. We will first consider the task, and describe its essential characteristics. We will then examine the activity developed in response to the task.

Structure of a task

The *task object* is that which is to be transformed or studied. Tasks involving material objects were originally the most studied by ergonomic psychology. Tasks for which the "objects" include human individuals (service professions, therapeutic work, teaching) or for which the goal is to learn and acquire tools for thought (being a student) require a more complex analysis. For the teacher, the goal to be

attained is often described in procedural terms, with action verbs: "Teach the concept of length measurement to elementary school students," "correct a math test," "follow the curriculum." Goals can also be stated with reference to the student-knowledge relationship: "Have the student acquire the concept of length and linear units," "Have the student represent functions as mathematical objects and tools." For the student, the task is defined by the teacher's statement, and the requirements of mathematical work.

Tasks and sub-tasks

In a complex situation, the goal to attain consists of various sub-goals, whose achievement order is more or less constrained. For example, "introduce students to the concept of functions," in ninth grade, involves making documentation choices, creating lesson plans that cover one or more class sessions, defining the student tasks, conducting the in-class activities, and finally evaluating students' acquired knowledge.

The structure of the task involves transitions between the intentions of the prescribed task and the actual task as implemented

In a workplace (in the teacher's case) or learning environment (in the student's case), the subject responds to tasks assigned by a prescriber, with the framework for completion defined by the desired results and the permitted resources. This constitutes a prescribed task. But an *activity* is not a direct response to a prescribed task. The task is first redefined by the subject. To complete this task, the subject must form a representation of the task, allowing or forbidding possibilities (not always consciously), lifting or imposing restrictions, and using evaluation criteria that may differ from those of the prescription. This constitutes the *effective task*, to which the subject's activity represents a response. Misunderstandings in teaching are an expression of differences between the task anticipated by the teacher, and the task responded to by the student.

The gap between prescribed and effective tasks is inherent to the existence of two viewpoints: That of the task prescriber, and that of the task completer. The task the subject completes can differ from the assigned task for various reasons: Because the subject lacks motivation to engage in the desired actions, because the subject lacks the necessary competencies, because the subject constructed an inappropriate representation of the task, or even because of a divergence between the intended and prescribed tasks. The effective task is revealed by the subject's activity.

Analysis of the activity

In work or training situations, activity is oriented towards the completion of the task. Observable actions that permit an analysis are, first, operations on the objects of action, regardless of the aim of the research. This explains why the analysis of the activity relies on a preliminary analysis of the task, which can be understood as a psychological task analysis (Vicente, 1999) that relies on domain expertise.

However, the activity includes more than simply actions on "what to do," and includes other personal factors. For example, a teacher can assign different sets of problems from one year to another, not only because of the effect on students, but also to maintain personal motivation and avoid repetition or fatigue.

The analysis of a student task requires a didactician's mathematic expertise in order to identify what a student can do to effectively complete it. This is the aim of the *a priori* analysis presented in Chapter 2. The analysis of the teacher's task is more delicate. It is a largely discretionary task for which there is no defined procedure to follow (Leplat, 1997, p. 21). How to identify a strategy that would lead to the desired goal remains an open question, as there is no commonly accepted definition of an "expert teacher."[3] For this analysis of teachers' tasks and activities, we refer to a model of teaching as management of a dynamic human environment (Rogalski, 2003), in which the teacher mediates (Wood, Bruner, & Ross, 1976) between the student and the knowledge to be acquired (Robert & Rogalski, 2005), and in which language plays a central role (Pariès, Robert, & Rogalski, 2005).

The subject and the situation

The theory of activity depends on two other key concepts: The subject and the situation. We are interested in an *individual subject*, who has intentions and competencies (potential resources and personal constraints). Within this framework, subjects do not identify with their role, even though they may be constrained by legal and other responsibilities that act on the teacher. We can look both for commonalities between subjects and for specific aspects of their activities: What factors and organizational aspects do they share? What are the individual differences between them?

Whether students or teachers, subjects are not the sole masters of their goals or methods. They act within a *work* or *training situation*, which consists of a system of resources and constraints. Within this system, the teacher completes a set of tasks, which we can more globally consider to be a mission (the discretionary dimension of the task), tied to a prescriber (employer, supervisor) by a partially implicit contract. The teacher is acting within a context where students encounter multiple interventions (parents, teachers of other subjects, etc.) and within a process that continues during students' entire schooling. The student's situation is not limited to the tasks prescribed by the teacher under a didactic contract, but includes the social and familial environment.

We will now present the model of double regulation of activity, which can be related to issues of learning and development, as defined through the theories of Piaget and Vygotsky, and expanded by Vergnaud. Later on, we will defend the complementarity of Piaget and Vygotsky.

The model of double regulation of activity

The concept of regulation reflects the fact that the activity modifies the state of the situation as much as the state of the actor. The situation is a determining factor of the activity, and is simultaneously itself modified by the activity. This modification primarily affects the object of the activity, but can also include modification of resources and constraints. Subjects, too, both determine the activity and are modified in turn by their own activity. The situation can affect their potential for knowledge and action (competencies), their physical state (tired, sleepy, etc.), or their emotional state (happy, bored, anxious, etc.).

Figure 1 presents a schematic diagram of how this system of double regulation relates to the system of situational and subject determinants.

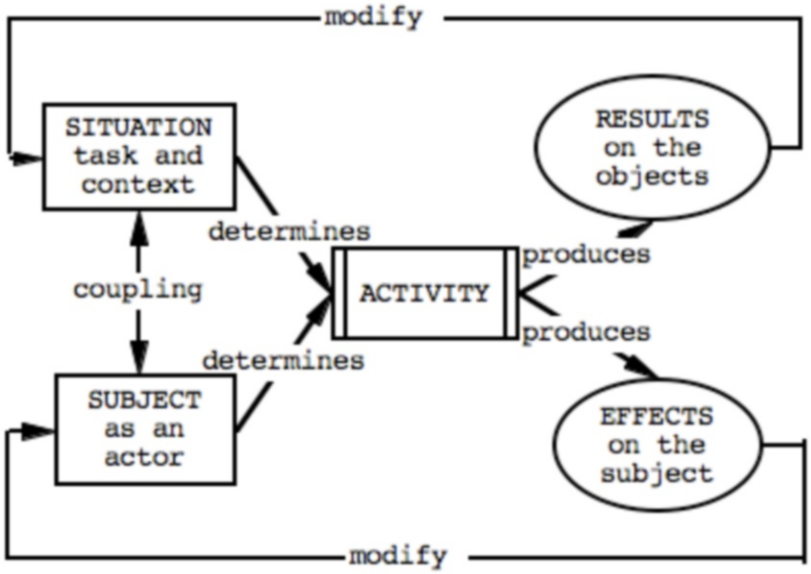

Figure 1. The double regulation includes a co-determination of the activity by situational and subject properties, as well as a double modification of the situation and the subject that is created by the results and effects produced (and by their agreement with expectations and acceptable outcomes).

This regulation can be considered in terms of short-term adjustments to action and "local" learning (such as learning how to find the inverse image of a function on a graph), or in terms of long-term development of a subject (understanding the concept of a function). The model of double regulation fits directly with the constructivist theories of Piaget and Vygotsky. It also sheds light on the issue of didactic intervention, by considering situational properties as potential producers of

learning and development. Before considering this further, we must first pause to define the specific activities of the teacher and student, respectively.

A framework for defining student and teacher activities

The student, whether as an individual, set of students, or class as a whole, is a central determining factor of the teacher's activity. The choice of lesson plans (student task organization), and the unfolding of these plans in class, depends on prior knowledge of students, as well as on the possible actions the teacher believes to be possible in class. The teacher's didactic interventions in class depend on students' individual or collective activity. Completing a task produces a return effect on the teacher's activity, with an eventual adjustment both of the proposed tasks and of the teacher's own activity. Students' behavior can also contribute to the effects on the teacher, inducing fatigue, enjoyment, etc.

The teacher determines the activity of the student through the assigned mathematical tasks. During the completion of a task, the teacher mediates between the students and the mathematical concept to be acquired. This mediation can consist of assistance in getting started, procedural or constructivist assistance in completing the task, evaluation of the final product, identification of the concept in play, etc. The teacher can also participate in the construction of a student's reflexivity (for example, by demonstrating how to solve problems) and intervene in the constructivist dimension of the student's activity. Chapter 2 will explore this question.

COMPARISON OF THE THEORIES OF PIAGET AND VYGOTSKY ON DEVELOPMENT AND LEARNING

Piaget and Vygotsky each elaborated theoretical frameworks for understanding children's (and, more generally, humans') developmental processes. We will first present each researcher's scientific objectives, then the relevant elements of Piagetian constructivism, and finally Vygotsky's theoretical contributions. Putting these two frameworks in perspective highlights their commonalities, which include factors of development, a long-term perspective, and the role of tools in development (called "cognitive tools" by Piaget and "psychological tools" by Vygotsky).

Piaget's and Vygotsky's scientific objectives

For Piaget, the crucial point, distinguishing Piaget from all others in the field and rendering him irreplaceable within the scientific panorama of the 20th century, is his objective of genetic epistemology. The central question of this is how humans acquire knowledge, and how they thereby progress from children to adults capable of contributing to the development of scientific knowledge.

Piaget's aim is to "try to untangle the roots of the diverse varieties of knowledge, beginning with their most elementary forms and following their

development to subsequent levels, including scientific thought" (Piaget, 2005, p.6). He notes that development of knowledge during the evolution of a species could in theory be considered part of this objective, but he chooses to begin with the development of a human child. He insists on the fact that his work has "a psychological dimension, but as a by-product … the goal is essentially epistemological" (op. cit., p. 7).

Piaget's viewpoint is therefore that of knowledge development for an *epistemological subject*, which is as much a theoretical construct as the *didactic subject* of mathematical didactics (when it defines teacher and student in terms of their role in the school system). A biologist by training, Piaget always insists on the biological roots of knowledge (Piaget, 1971, 2005, pp. 59-75).

Piaget's interest in the evolution of the structures of knowledge leads him to neglect a certain number of topics. For example, the topic of the developmental factors of a child (considered as a *psychological subject*) will not be central to Piaget's work. This is not because Piaget denies the effects of factors that are not internal to the "epistemological subject," but because his objective is to understand the internal process of development.

Piagetian constructivism claims that knowledge of objects is constructed through actions on these objects, and Piaget's goal is to demonstrate his approach's validity on the set of large domains of knowledge. These actions are not limited to physical acts on material objects, as knowledge construction can also occur through mental operations. Observation, for example, is a valid action that affects a subject's representations.

As for Vygotsky, his goal of theorization is clearly psychological, aiming to theorize the "higher functions" of thought. For him, the subject is a psychological subject, considered from the beginning to be in a social interaction with other subjects who have previously and personally developed "psychological tools" : this enables the development of knowledge. Under this model, knowledge of the world is socially preexisting in children: Their cognitive activities exist within social interactions before they are internalized into a subjective plane. This is the central, and very strong, idea of socio-constructivism: The passage from the inter-individual to intra-individual relies on the construction of psychological tools.

Vygotsky's focus is therefore profoundly different from Piaget's, with completely different objectives. Vygotsky's subject is an individual and social subject, who will construct tools for thought within social interactions. Piaget's subject is an epistemological subject, for whom the organization of knowledge (rather than mediation or tools) is the issue.

From this starting point, Vygotsky describes in a theoretical fashion the processes of learning and development, without dissociating the two. He will particularly differentiate, within a subject's "learning-development," the "everyday" concepts from the "scientific" concepts. Everyday concepts come from the everyday world, where social interactions do not have as a goal the production of an organized conceptual piece of knowledge in children. The acquisition of scientific concepts is accomplished through deliberate didactic interventions (Vygotsky, 1986, chap. 6).

Under Vygotsky's theory, scientific concepts are taught in scholastic institutions, and develop differently from everyday concepts. This deeper theoretical understanding of the evolution of concepts is directly pertinent for all didactics of a knowledge domain.

We highlight these differences between Piaget's and Vygotsky's objectives and central objects as a preface to presenting evidence that their case is not one of two psychologists with conflicting viewpoints, but rather of each making his own specific scientific contribution. Each proposes an original perspective on knowledge construction, as defended by Shayer (2003), for example. We will therefore first go deeper into the framework of Piagetian constructivism, and then describe the theoretical contributions of Vygotskian conceptualism, which are crucial for didactics of science.

Piagetian constructivism

The dominant image of Piagetian constructivism is probably that of a construction of knowledge that is internal to the subject. From this, one could see Vygotskian socio-constructivism as in opposition, taking into account the social dimension that Piaget would supposedly discard. To show that this is simply a question of perspective, we can refer to Piaget himself: "The social group plays … from a cognitive point of view the same role that the 'population' plays from a genetic point of view. … In this sense the society is the supreme unit and the individual only achieves his intellectual constructions insofar as he is the seat of collective interactions for which the level [depends] on the society as a whole" (Piaget, 1992, p. 345). The necessity of the social aspect in cognitive development is here clearly affirmed. Piaget successfully integrates the existence of two shifts during development: One associated with the individual as epistemological subject, and the other purely social. But it is the process of organization of knowledge (its structure) that will be central in the research he conducts. This "internal mechanism" of development is conceived in terms of a double regulation, retroactive and proactive,[4] for which Figure 2 presents a schematic diagram.

We can consider this double loop as a "zoomed-in" portion of the system of activity regulation (Figure 1). The object of the action is what is retained in the situation: The comparison between the intended state of this object and the observed effect releases an adjustment of the action. The feedback on the subject (which was not made explicit in Figure 1) will modify the action "upstream" through an adaptation of knowledge and schemes for action. Moreover, inasmuch as there is an intended or anticipated result, the action is also regulated proactively ("feedforward"). Piaget defines this mechanism in terms of a dialectic between *assimilation* of the new situation into the subject's strategies and conceptualizations, and *accommodation* of these concepts and of their organization. (We can think of the passage from a one-dimensional treatment of objects to a bi-dimensional treatment, for which the model is the Cartesian product.)