Noise and Nonlinear Phenomena in Nuclear Systems

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Noise and Nonlinear Phenomena in Nuclear Systems

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This book is dedicated to Professor Vicente Serradell, who founded the Department of Nuclear Engineering at the Polytechnical University of Valencia. His continuous efforts made possible the formation of a generation of professors in the nuclear sciences, who are now working actively in this and other areas of research.

- Basic methodology to analyze nonlinear deterministic and stochastic processes.

- Deterministic analysis of nuclear dynamics, transition to chaos.

- Noise theory and its application to the surveillance and diagnosis of nuclear systems.

The presentations more relevant from an engineering point of view were related to the use of stochastic methods to monitor nuclear systems and the application of recent developments in nonlinear dynamics to xenon oscillations, control theory, limit cycle analysis, breeding of fertile isotopes, analysis of bifurcations, and jumping processes in nuclear reactors.

Several presentations were dedicated to the discussion of the use of the third moment as a stochastic descriptor, with experimental applications to criticality safety, safeguard, and enhancement of the sensitivity of the monitoring system to nonlinear processes. The theoretical background of the noise techniques in both the time and frequency domains were also discussed; a general theory of space- and energy-dependent stochastic processes in zero power systems was presented, illustrating that the Boltzmann equation for neutrons is only the first moment of a more general formalism. Complications associated with this general formalism make very useful the discussion of detailed solutions of simplified models like the explicit and implicit solutions of the forward and backward stochastic equations with and without including the burn-up in the case of "point" systems. Specific noise techniques like the method of the ^{252}Cf source were analyzed with a very general model that includes space and energy effects.

Particularly relevant to the design of high-intensity neutron sources was the application of nonlinear analysis to the problem of xenon oscillations after coupling the kinetic and conservation equations with the equation for the reactivity. The evolution of the nuclear species in breeding assemblies was presented in the light of the theory of dynamical systems. Recent developments in the control of nuclear reactors using the Pontryagin maximum principle to a nonlinear

PREFACE

The main goal of the meeting was to facilitate and encourage the application of recent developments in the physical and mathematical sciences to the analysis of deterministic and stochastic processes in nuclear engineering. In contrast with the rapid growth (triggered by computer developments) of nonlinear analysis in other branches of the physical sciences, the theoretical analysis of nuclear reactors is still based on linearized models of the neutronics and thermal-hydraulic feedback loop, an approach that ignores some intrinsic nonlinearities of the real system. The subject of noise was added because of the importance of the noise technique in detecting abnormalities associated with perturbations of sufficient amplitude to generate nonlinear processes.

Consequently the organizers of the meeting invited a group of leading researchers in the field of noise and nonlinear phenomena in nuclear systems to report on recent advances in their area of research. A selected subgroup of researchers in areas outside the reactor field provided enlightenment on new theoretical developments of immediate relevance to nuclear dynamics theory.

Thirty five presentations were discussed by participants from fourteen countries and international organizations in the pleasant environment provided by the Polytechnical University of Valencia. The logistic support and the active help of the University was crucial to the success of the meeting. The organizers were pleased to witness a cross-fertilization of ideas between researchers: for example new stochastic tools to detect nonlinear processes in nuclear power plants are being developed, whereas the paradigm of noise perturbations in a distorted potential well is being applied to analyze jumping processes in the internal vibrations of nuclear reactors. The papers presented at the meeting fall into some of the following categories:

reactor dynamics model were discussed; because the method allows parameter tracking it could be used for the on-line surveillance of nuclear systems.

One of the finer presentations from a theoretical point of view was the application of the Hartman-Grobman theorem to the analysis of limit cycles observed in boiling water reactors that allows the parametrization of the condition under which a limit cycle appears. Particularly interesting because of its novelty was the presentation about jumping processes in bistable systems subject to noisy perturbations; the paradigm of a double potential well was used to analyze the noise signatures of the vibrations of reactor components such as ,for example, the pressure vessel. Results obtained recently on strongly nonlinear phenomena relevant to nuclear science and technology were summarized.

There was a general concensus that the workshop had been very useful and that the experience should be repeated some years hence. There are already preliminary plans for a further meeting in 1990.

The Organizing Committee gratefully acknowledges the financial support of the following institutions: Scientific Affairs Division of NATO, Direccción General de Investigación Científica y Técnica of Spain, and the Polytechnical University of Valencia. Their contributions made this workshop possible. The editors also acknowledge the help of Ms. Lucía Ferreres in the preparation of this book.

J.L. Muñoz-Cobo
Felix C. Difilippo
Valencia, Spain, July 26, 1988

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VII. Nonlinear Dynamics and Transition to Chaos in Nuclear Systems. Part II

CHAPTER I

INTRODUCTION: BASIC CONCEPTS

A GLIMPSE INTO THE WORLD OF

RANDOM WALKS

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ABSTRACT

Some classical results from random walk theory are reviewed. Asymptotic properties are derived for random walk on a one-dimensional lattice with static disorder. Fractal time properties are illustrated on a simple example.

INTRODUCTION

It would be impossible, both for reasons of space and competence, to write a comprehensive review on random walks. Besides, many excellent reviews and books have been published on this topic [1-10]. Our purpose here will be to collect a few simple illustrative results, which may hopefully inspire ideas for applications in the field of noise in nuclear reactors.

The concept of "random walk" is used in a broad sense of a stochastic walk on a set M of discrete states $\underset{\sim}{m} \in M$. This, for instance, encompasses Markovian walks such as birth and death processes. The walk is characterized by the transition rules between the states and the geometry of the set M. The mathematical structure of the random walk problem is, in fact, closely related to other physical problems, some of which are discussed briefly in §2. In particular, a Fokker-Planck or Langevin description is included as a limiting case.

A central quantity for stationary random walks is the conditional probability $P(\underset{\sim}{m}|\underset{\sim}{m}_o,t)$ to go from site $\underset{\sim}{m}_o$ to site $\underset{\sim}{m}$ during a time t, as well as its Laplace transform, the so-called Greens function of the walk :

$$\tilde{P}(\underset{\sim}{m}|\underset{\sim}{m}_o,s) = \int_o^\infty dt\, e^{-st}\, P(\underset{\sim}{m}|\underset{\sim}{m}_o,t) \tag{1}$$

Some results for $\tilde{P}$ will be reviewed in §3. Furthermore, it is discussed how various other important quantities can be derived from it.

A lot of recent work on random walks is dealing with the study of anomalous behavior. By this we mean a qualitative departure of various random walk properties (such as mean square displacement and spectral properties) from their standard from (as obtained for Markovian walks on Eucledian lattices). In this context, we derive results for random walks with static disorder in §4 and discuss the case of random walks with fractal aspects in §5. Transport properties in random media have been discussed in the context of reactor noise [12]. The concept of fractal structures however, is probably new but may, in view of its universality, find some application in this field.

RANDOM WALK AND RELATED PROBLEMS

In the case of a Markovian random walk, the conditional probability $P(\underset{\sim}{m}|\underset{\sim}{m}_o,t)$ obeys the so-called Master Equation :

$$\partial_t P(\underset{\sim}{m},t) = \sum_{\underset{\sim}{m}} [W(\underset{\sim}{m}|\underset{\sim}{m}') P(m',t) - W(\underset{\sim}{m}'|\underset{\sim}{m}) P(\underset{\sim}{m},t)] \qquad (2)$$

$W(\underset{\sim}{m}|\underset{\sim}{m}')$ stands for the transition probability, per unit time, to go from $\underset{\sim}{m}'$ to $\underset{\sim}{m}$. Note that equation (2) is linear in P : the different random walkers do not interact. "Nonlinearity" or "collisions" can however be included in a mean field way by considering a nonlinear dependence of the transition probability on the state of the system. Nonlinear birth - and death processes are of this type : a neutron, that is produced, is added to the bath of all existing neutrons, and each individual neutron is subsequently interacting with this bath.

Equation (2) has an electrical and mechanical analogue (see Fig. 1) [5]. In the electrical system, the electrical potential $V(\underset{\sim}{m},t)$ obeys the following equation :

$$\partial_t V(\underset{\sim}{m},t) = \sum_{\underset{\sim}{m}'} \frac{(V(\underset{\sim}{m}',t) - V(\underset{\sim}{m},t))}{R(\underset{\sim}{m},\underset{\sim}{m}')C(\underset{\sim}{m})} \qquad (3)$$

where $C(\underset{\sim}{m})$ is the capacitance linking site $\underset{\sim}{m}$ to the earth and $R(\underset{\sim}{m},\underset{\sim}{m}')$ is the resistance between sites $\underset{\sim}{m}$ and $\underset{\sim}{m}'$. In the mechanical analogue, one considers a distribution of harmonically interconnected masses in a plane. The spring constant between sites $\underset{\sim}{m}$ and $\underset{\sim}{m}'$ is denoted by $k(\underset{\sim}{m},\underset{\sim}{m}')$. The small deviations $x(\underset{\sim}{m},t)$, orthogonal to the plain, obey the following set of Newton equations :

$$\partial_t^2 x(\underset{\sim}{m},t) = \sum_{\underset{\sim}{m}'} k(m,m') [x(\underset{\sim}{m}',t) - x(\underset{\sim}{m},t)] \qquad (4)$$

As an illustrative example of a quantum-mechanical problem that is described by an equation of the same form, we cite the Kronig-Penney model of a particle (e.g. an electron) in a one-dimensional periodic delta function potential. If we call $\psi(m)$ the value of an energy eigenfunction for the energy equal to E at the right hand side of the

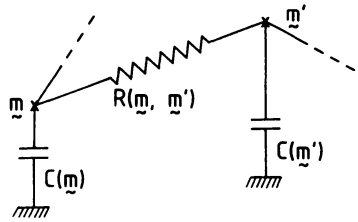

Fig. 1a. Electrical Analogue

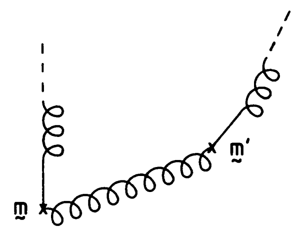

Fig. 1b. Mechanical Analogue

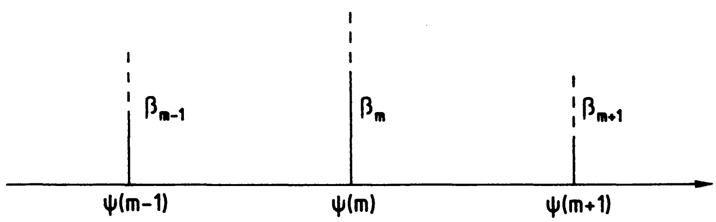

Fig. 1c. Quantum-Mechanical Analogue

Figure 1 . Electrical, Mechanical and Quantum-Mechanical Analogue of the Random Walk Master Equation

potential peak m, then an easy calculation shows that the Schrödinger equation implies the following set of equations [12] :

$$2(\cos k + \beta_m \frac{\sin k}{2k} - 1)\psi(m) = \psi(m+1) + \psi(m-1) - 2\,\psi(m) \tag{5}$$

where β_m is the weight of the potential peak at m, $k = a(2m\,E/\hbar)^{1/2}$ and a is the lattice spacing. We have written the right hand side such that it has the same form as the right hand side of (2) for a symmetric nearest neighbour random walk.

Finally, as already mentioned in the introduction, equation (2) is more general then a Fokker-Planck description since the latter can be obtained in the appropriate limit. A disturbing discrepancy subsists in the thermodynamic limit of systems with extensive transition probabilities in the region of multiple stable states [14]. Moreover, (2) is certainly a more natural description if one is concerned with internal fluctuations, i.e. the fluctuations that arise from the very dynamics of the system. Also, the Ito versus Stratonovich (or other) interpretations of Langevin equations with multiplicative noise reflect the fact that relevant information on the interplay between noise and fast dynamical variables may have been discarded [6,13].

THE CONDITIONAL PROBABILITY AND RELATED QUANTITIES

Once the explicit solution of equation (2) has been found, a lot of interesting random walk properties become available. Usually, it is easier to obtain an explicit compact form for the Laplace transform (1). A list of results for Markovian nearest neighbour random walks in 1 dimension is compiled in table I. In the following, we restrict ourselves to stationary Markov processes (W in equation (2) is time-independent).

Let us call $F(\underset{\sim}{m},\underset{\sim}{m}_o,t)$ the first passage time density to reach $\underset{\sim}{m}$ starting from $\underset{\sim}{m}_o$ after a time t. The following "renewal equation" states that to go from $\underset{\sim}{m}_o$ to $\underset{\sim}{m}$ after a time t, one must have reached $\underset{\sim}{m}$ (for the first time) at an earlier time τ [1] :

$$P(\underset{\sim}{m}|\,\underset{\sim}{m}_o,t) = \int_o^t P(\underset{\sim}{m}|\,\underset{\sim}{m},t-\tau)\;F(\underset{\sim}{m}|\,\underset{\sim}{m}_o,\tau)\;d\tau \tag{6}$$

By Laplace transformation, one finds [1] :

$$\tilde{F}(\underset{\sim}{m}|\,\underset{\sim}{m}_o,s) = \frac{\tilde{P}(\underset{\sim}{m}|\,\underset{\sim}{m}_o,s)}{\tilde{P}(\underset{\sim}{m}|\,\underset{\sim}{m},s)} \tag{7}$$

For a one-dimensional nearest-neighbour random walk, the following stronger result, due to Siegert [15], holds true :

$$\tilde{F}(m|\,m_o,s) = \frac{\tilde{P}(m'\,|\,m_o,s)}{\tilde{P}(m'\,|\,m,s)} \qquad m_o \le m \le m' \tag{8}$$

The moments T_n of the first passage time are given by the following result :

$$T_n(\underset{\sim}{m}|\underset{\sim}{m}_o) = \int_o^{\infty} t^n F(\underset{\sim}{m}|\underset{\sim}{m}_o,t)\, dt = (-1)^n \frac{\partial^n}{\partial s^n} \tilde{F}(\underset{\sim}{m}|\underset{\sim}{m}_o,s)\Big|_{s=0} \tag{9}$$

This implies that the limiting behavior of $\tilde{F}$ hence of $\tilde{P}$ for $s \to 0$, is sufficient to evaluate the moments T_n. This limiting behavior is available for the case of a nearest neighbour random walk in one dimension and one finds (for $m > m_o$) :

$$T_n\,(m|m_o) = n \sum_{r=m_o}^{m-1} \sum_{s=-\infty}^{r} \frac{T_{n-1}(m|s)\,P(s)}{W(r|r+1)P(r)} \tag{10.a}$$

with

$$T_o\,(m|s) = 1 \tag{10.b}$$

and $P(s)$ is the steady state probability for the random walk with transition rates W and a reflecting boundary condition at $s = m$, $W(m|m-1) = 0$.
The result for $n = 1$ is well known [17], and in the limit of Fokker-Planck dynamics, the results of reference [18] are recovered.
These results can also be generalized to the case of first passage time statistics to more then one final destination [19].

The average number of distinct sites (or span) $S(\underset{\sim}{m}_o,t)$ visited in a random walk starting at $\underset{\sim}{m}_o$ after a time t is another important quantity. $S(\underset{\sim}{m}_o,t)$ can be written as follows [1] :

$$S(\underset{\sim}{m}_o,t) = \sum_{\underset{\sim}{m}} < \theta(\underset{\sim}{m},t) > \tag{11}$$

where $\theta(\underset{\sim}{m},t)$ is a random variable which is equal to 1 if site $\underset{\sim}{m}$ has been visited before t, i.e. if the first passage time is smaller then t, and zero otherwise. Obviously, one has :

$$<\theta(\underset{\sim}{m},t)> = \int_o^t F(\underset{\sim}{m}|\underset{\sim}{m}_o,\tau)\, d\tau \tag{12}$$

From (11) and (12), one finds [1], for a translationally invariant system :

$$\begin{aligned}\tilde{S}(\underset{\sim}{m}_o,s) &= \int_o^{\infty} e^{-st}\, S(\underset{\sim}{m}_o,t)dt \\ &= \sum_{\underset{\sim}{m}} \frac{\tilde{F}(\underset{\sim}{m}|\underset{\sim}{m}_o,s)}{s} = \frac{1}{s^2\tilde{P}(\underset{\sim}{m}_o|\underset{\sim}{m}_o,s)}\end{aligned} \tag{13}$$

where we have made use of equation (7). We conclude that the span is intimately linked to the probability of return to the origin.

A situation occurring in a large variety of physical problems is that of a particle with a random velocity u describing the rate of change of a coordinate x. More precisely, we suppose that u depends on an internal state $\underset{\sim}{m}$, $u = u(\underset{\sim}{m})$, which is undergoing a random walk on a state space M. The equations of motion are thus :