Everyday Heat Transfer Problems

Sensitivities To Governing Variables

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INTRODUCTION

Everyday engineering problems in heat transfer can be very complicated and may require solutions using finite element or finite difference techniques in transient mode and in multiple dimensions. These engineering problems might cover conduction, convection and radiation energy transfer mechanisms. The thermophysical properties that govern a particular heat transfer problem can be challenging to discover, to say the least.

Some of the standard thermophysical properties needed to solve a heat transfer problem are density, specific heat at constant pressure, thermal conductivity, viscosity, volumetric thermal expansion coefficient, heat of vaporization, surface tension, emissivity, absorptivity, and transmissivity. These thermophysical properties can be strong functions of temperature, pressure, surface roughness, wavelength and other properties. in the region of interest.

Once a heat transfer problem's assumptions are made, equations set up and boundary conditions determined, one should investigate the sensitivities of desired outputs to all the governing independent variables. Since these sensitivities are mostly non-linear, one should analyze them in the region of interest. The results of such sensitivity analyses will provide important information as to which independent variables should be researched thoroughly, determined accurately, and focused on. The sensitivity analysis will also provide insight into uncertainty analysis for the dependent variable, (Reference S. J. Kline and F. A. McClintock [9]). If the dependent variable y is defined as a function of independent variables $x_1, x_2, x_3, \ldots x_n$ as follows:

$$y = f(x_1, x_2, x_3, \ldots x_n)$$

then the uncertainty U for the dependent variable can be written as:

$$U = [(\partial y/\partial x_1\ u_1)^2 + (\partial y/\partial x_2\ u_2)^2 + (\partial y/\partial x_3\ u_3)^2 + \ldots + (\partial y/\partial x_n\ u_n)^2]^{0.5}$$

where $\partial y/\partial x_1, \partial y/\partial x_2, \partial y/\partial x_3, \ldots, \partial y/\partial x_n$ are the sensitivities of the dependent variable to each independent variable and $u_1, u_2, u_3, \ldots, u_n$ are the uncertainties in each independent variable for a desired confidence limit.

In this book, I will provide sensitivity analyses to well-known everyday heat transfer problems, determining $\partial y/\partial x_1, \partial y/\partial x_2, \partial y/\partial x_3, \ldots, \partial y/\partial x_n$ for each case. The analysis for each problem will narrow the field of independent variables that should be focused on during the design process. Since most heat transfer problems are non-linear, the results presented here would be applicable only in the region of values assumed for independent variables. For the uncertainties of independent variables—for example, experimental measurements of thermophysical properties—the reader can find the appropriate uncertainty value for a desired confidence limit within existing literature on the topic.

Each chapter will analyze a different one-dimensional heat transfer problem. These problems will vary from determining the maximum allowable current in an open-air electrical wire to cooking a turkey in a convection oven. The equations and boundary conditions for each problem will be provided, but the focus will be on the sensitivity of the governing dependant variable on the changing independent variables. For the derivation of the fundamental heat transfer equations and for insight into the appropriate boundary conditions, the reader should refer to the heat transfer fundamentals books listed in the references.

Problems in Chapters 1 through 6 deal with steady-state and one-dimensional heat transfer mechanisms in rectangular coordinates. Chapters 7 through 10 deal with steady-state and one-dimensional heat transfer mechanisms in cylindrical coordinates. Unsteady-state problems in one-dimensional rectangular coordinates will be tackled in Chapters 11 through 14, cylindrical coordinates in Chapter 15, and spherical coordinates in Chapters 16 through 18.

The following six chapters are allocated to special heat transfer problems. Chapters 19 and 20 deal with momentum, mass and heat transfer analogies used to solve the problems. Chapter 21 analyzes a counterflow heat exchanger using the log mean temperature difference method. Chapters 22 and 23 solve heat transfer problems of ice formation and solidification with moving boundary conditions. Chapter 24 analyzes the problem of frictional heating of materials in contact with moving sources of heat.

I would like to thank my engineering colleagues G. W. Hodge, A. Z. Basbuyuk, E. O. Atesmen, and S. S. Tukel for reviewing some of the chapters. I would also like to dedicate this book to my excellent teachers and mentors in heat transfer at several universities and organizations. Some of the names at the top of a long list are Prof. W. M. Kays, Prof. A. L. London, Prof. R. D. Haberstroh, Prof. L. V. Baldwin, and Prof. T. N. Veziroglu.

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CHAPTER 1

HEAT LOSS FROM WALLS IN A TYPICAL HOUSE

Heat loss from the vertical walls of a house is analyzed under steady-state conditions. Walls are assumed to be large and built in a planar fashion, so that one-dimensional heat transfer rate equations in rectangular coordinates may be used, and only conduction and convection heat transfer mechanisms are considered. In this analysis, radiation heat transfer effects are neglected. No air leakage through the wall was assumed. Also, the wall material thermal conductivities are assumed to be independent of temperature in the region of operation.

Assuming winter conditions—the temperature inside the house is higher than the temperature outside the house—the convection heat transferred from the inside of the house to the inner surface of the inner wall is:

$$Q/A = h_{in}\ (T_{in} - T_{\text{inner wall inside surface}}) \qquad (1\text{-}1)$$

Most walls are constructed from three types of materials: inner wall board, insulation and outer wall board. The heat transfer from these wall layers will occur by conduction, and is presented by the following rate Eqs., (1-2) through (1-4):

$$Q/A = (k_{\text{inner wall}}/t_{\text{inner wall}})\ (T_{\text{inner wall inside surface}} - T_{\text{inner wall outside surface}}) \quad (1\text{-}2)$$

$$Q/A = (k_{\text{insulation}}/t_{\text{insulation}})\ (T_{\text{inner wall outside surface}} - T_{\text{outer wall inside surface}}) \quad (1\text{-}3)$$

$$Q/A = (k_{\text{outer wall}}/t_{\text{outer wall}})\ (T_{\text{outer wall inside surface}} - T_{\text{outer wall outside surface}}) \quad (1\text{-}4)$$

The heat transfer from the outer surface of the outer wall to the atmosphere is by convection and can be expressed by the following rate Eq. (1-5):

$$Q/A = h_{\text{out}}\ (T_{\text{out}} - T_{\text{outer wall outer surface}}) \quad (1\text{-}5)$$

Eliminating all the wall temperatures from Eqs. (1-1) through (1-5), the heat loss from a house wall can be rewritten as:

$$Q/A = (T_{\text{in}} - T_{\text{out}})/[(1/h_{\text{in}}) + (t_{\text{inner wall}}/k_{\text{inner wall}}) + (t_{\text{insulation}}/k_{\text{insulation}}) + (t_{\text{outer wall}}/k_{\text{outer wall}}) + (1/h_{\text{out}})] \quad (1\text{-}6)$$

The denominator in Eq. (1-6) represents all the thermal resistances between the inside of the house and the atmosphere, and they are in series.

In the construction industry, wall materials are rated with their R-value, namely the thermal conduction resistance of one-inch material. R-value dimensions are given as (hr-ft^2-F/BTU)(1/in). The sensitivity analysis will be done in the English system of units rather than the International System (SI units). The governing Eq. (1-6) for heat loss from a house wall can be rewritten in terms of R-values as follows:

$$Q/A = (T_{\text{in}} - T_{\text{out}})/[(1/h_{\text{in}}) + R_{\text{inner wall}}\ t_{\text{inner wall}} + R_{\text{insulation}}\ t_{\text{insulation}} + R_{\text{outer wall}}\ t_{\text{outer wall}} + (1/h_{\text{out}})] \quad (1\text{-}7)$$

where the definitions of the variables with their assumed nominal values for the present sensitivity analysis are given as:

Q/A = heat loss through the wall due to convection and conduction in Btu/hr-ft^2

T_{in} = 68°F (inside temperature)
T_{out} = 32°F (outside temperature)
h_{in} = 5 BTU/hr-ft^2-F (inside convection heat transfer coefficient)
$R_{inner\ wall}$ = 0.85 hr-ft^2-F/BTU-in (wall inside board R-value)
$t_{inner\ wall}$ = 1 in (wall inside board thickness)
$R_{insulation}$ = 3.5 hr-ft^2-F/BTU-in (insulation layer R-value)
$t_{insulation}$ = 4 in (insulation layer thickness)
$R_{outer\ wall}$ = 5 hr-ft^2-F/BTU-in (wall outside board R-value)
$t_{outer\ wall}$ = 1 in (wall outside board thickness)
h_{out} = 10 BTU/hr-ft^2-F (outside convection heat transfer coefficient).

The heat loss through a wall due to changes in convection heat transfer is presented in Figures 1-1 and 1-2. Changes in the convection heat transfer coefficient affect the heat loss mainly in the natural convection regime. As the convection heat transfer coefficient increases into the forced convection regime, heat loss value asymptotes. Resistances from both inside and outside convection heat transfer are too small to cause any change in heat loss through the wall.

The heat loss through a wall due to changes in insulation material R-value is presented in Figures 1-3 and 1-4. Higher R-value insulation

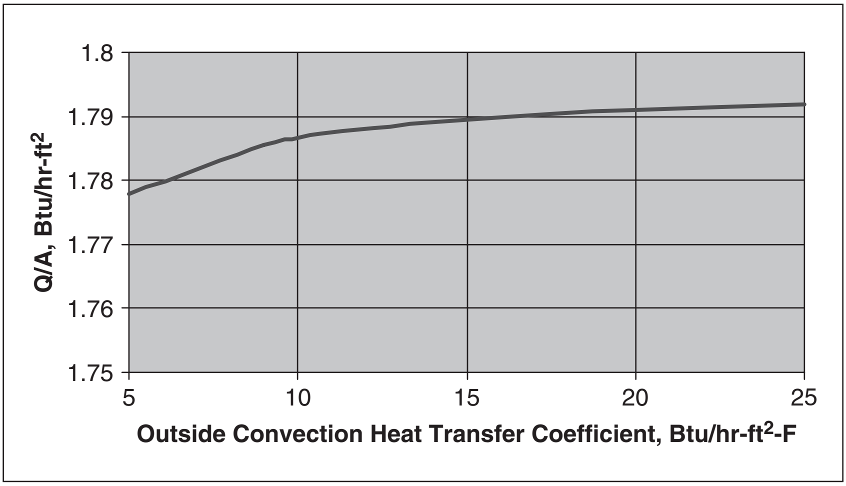

Figure 1-1 Wall heat loss versus outside convection heat transfer coefficient

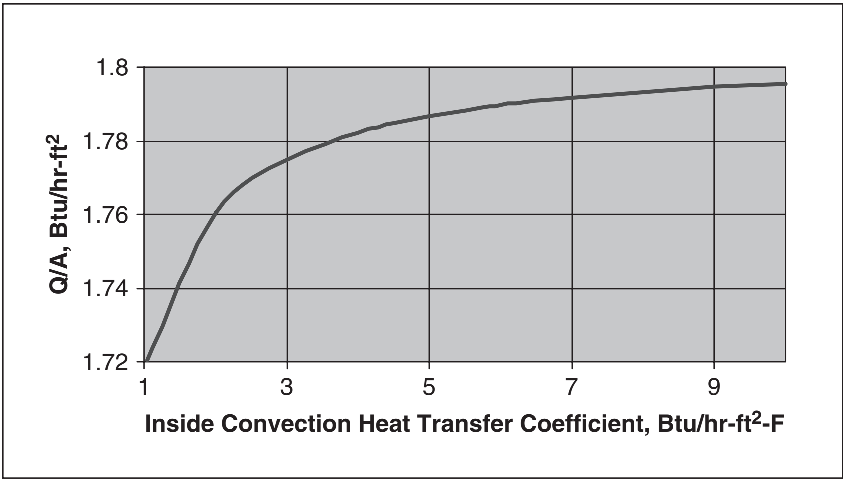

Figure 1-2 Wall heat loss versus inside convection heat transfer coefficient

material is definitely the way to go, depending upon the cost and benefit analysis results. The thickness of the insulation material is also very crucial. Thicker insulation material is definitely the best choice, depending upon the cost and benefit analysis results.

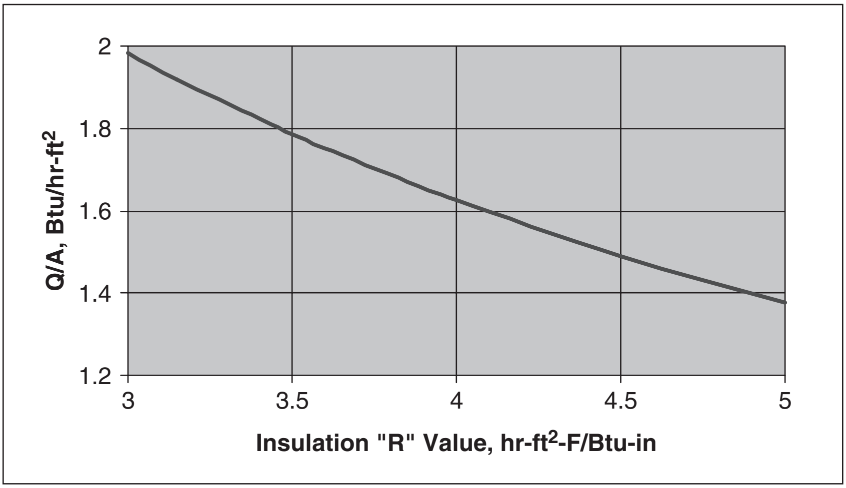

Figure 1-3 Wall heat loss versus insulation R-value

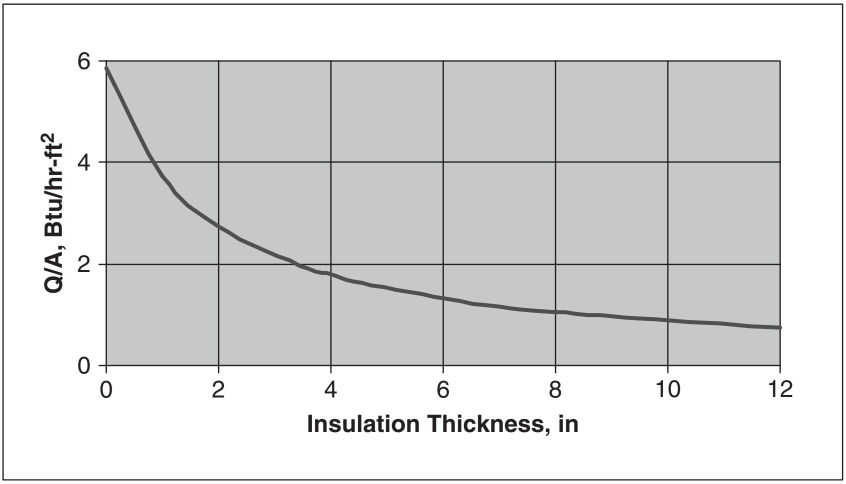

Figure 1-4 Wall heat loss versus insulation thickness

The effects on heat loss of inner and outer wall board R-values and thicknesses are similar to the effects of insulation R-value and thickness, but to a lesser extent. Sensitivities of heat loss to all the governing variables around the nominal values given above will be analyzed later.

Sensitivity of heat loss to the outside convection heat transfer coefficient can be determined in a closed form by differentiating the heat loss Eq. (1-7) with respect to h_{out}:

$$\partial(Q/A)/\partial h_{out} = (T_{in} - T_{out})/\{h^2_{out}\,[(1/h_{in}) + R_{inner\ wall}\,t_{inner\ wall} + R_{insulation}\,t_{insulation} + R_{outer\ wall}\,t_{outer\ wall} + (1/h_{out})]^2\} \qquad (1\text{-}8)$$

Sensitivity of heat loss to the outside convection heat transfer coefficient is given in Figure 1-5. Similar sensitivity is experienced for the inside convection heat transfer coefficient. The sensitivity of heat loss to the convection heat transfer coefficient is high in the natural convection regime, and it diminishes in the forced convection regime.

Sensitivities of heat loss to insulation material R-value and insulation thickness are given in Figures 1-6 and 1-7 respectively.

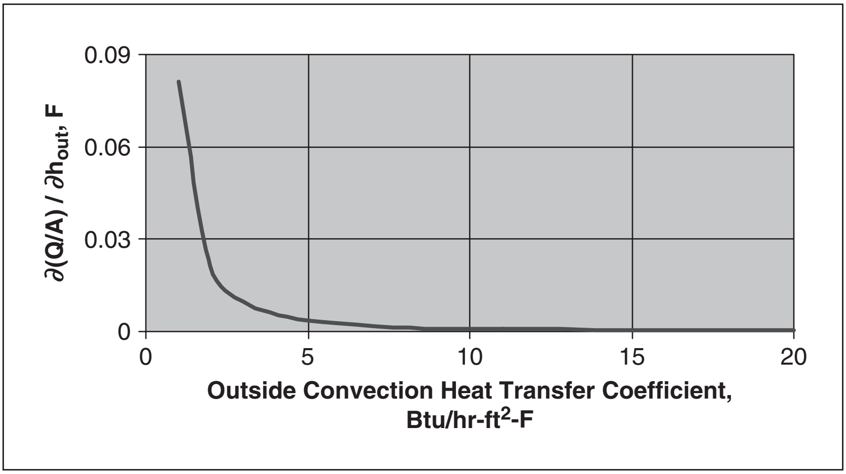

Figure 1-5 Sensitivity of house wall heat loss per unit area to the outside convection heat transfer coefficient

These two sensitivities are similar, as can be expected, since the linear product of insulation material R-value and insulation thickness affects the heat loss, as shown in the governing heat loss Eq. (1-7). Absolute sensitivity values are high at the low values of insulation

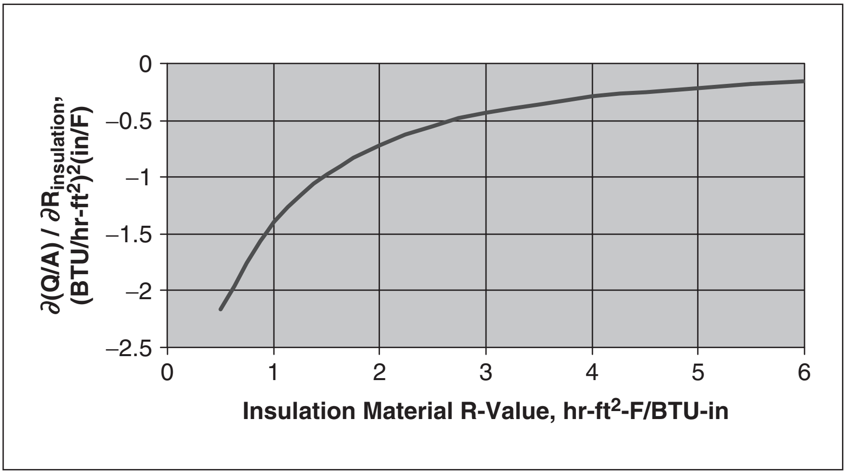

Figure 1-6 Sensitivity of house wall heat loss per unit area to insulation material R-value

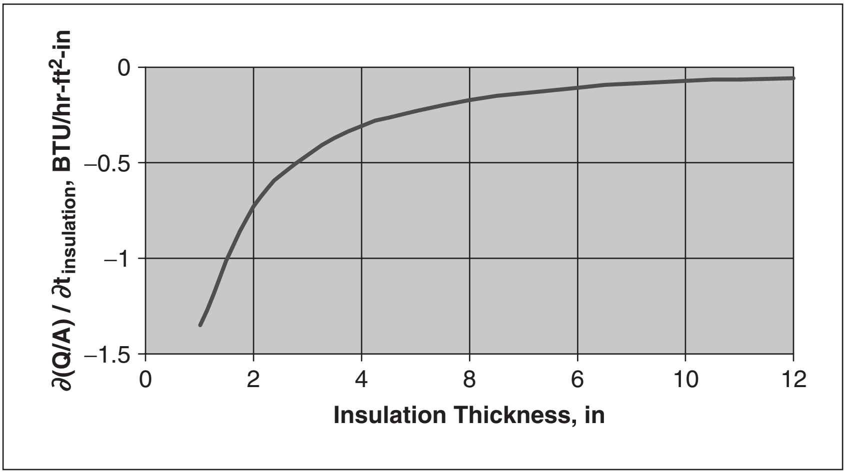

Figure 1-7 Sensitivity of house wall heat loss per unit area to insulation thickness

material R-value and insulation thickness. Sensitivities approach zero as insulation material R-value and insulation thickness values increase.

A ten-percent variation in independent variables around the nominal values given above produces the sensitivity results given in Table 1-1 The sensitivity results are given in a descending order and they are applicable only in the region of assigned nominal values, due to their non-linear effect to heat loss. The one exception is temperature potential, $(T_{in} - T_{out})$, which will always be ±10% due to its linear effect on heat loss. Material R-value and its thickness change have the same sensitivity, since their linear product affects the governing heat loss equation.

Heat loss through the wall is most sensitive to the temperature potential between the inside and outside of the house. Changes in wall insulation R-value and thickness affect heat loss as much as the temperature potential. Continuing in order of sensitivity, wall outer board R-value and thickness changes affect heat loss the most, followed by wall inside board R-value and thickness. Wall heat loss is

Table 1-1 House wall heat loss change per unit area due to a 10% change in variables nominal values

Variable	Nominal Value	House Wall Heat Loss Change Due To A 10% Decrease In Nominal Value	House Wall Heat Loss Change Due To A 10% Increase In Nominal Value
$T_{in} - T_{out}$	36°F	–10%	+10%
$R_{insulation}$	3.5 hr-ft^2-F/BTU-in	+7.467%	–6.497%
$t_{insulation}$	4 in	+7.467%	–6.497%
$R_{outer\ wall}$	5 hr-ft^2-F/BTU-in	+2.545%	–2.545%
$t_{outer\ wall}$	1 in	+2.545%	–2.545%
$R_{inner\ wall}$	0.85 hr-ft^2-F/BTU-in	+0.424%	–0.424%
$t_{inner\ wall}$	1 in	+0.424%	–0.424%
h_{in}	5 BTU/hr-ft^2-F	–0.110%	+0.090%
h_{out}	10 BTU/hr-ft^2-F	–0.055%	+0.045%

least sensitive to both the inside and outside heat transfer coefficient changes. Wall heat loss sensitivity to both the inside and outside heat transfer coefficient changes is an order of magnitude less than sensitivity to temperature potential changes.

CHAPTER 2

CONDUCTION HEAT TRANSFER IN A PRINTED CIRCUIT BOARD

Conduction heat transfer in printed circuit boards (PCBs) has been studied extensively in literature i.e., B. Guenin [4]. The layered structure of a printed circuit board is treated using two different thermal conductivities; one is in-plane thermal conductivity and the other is through-thickness thermal conductivity. One-dimensional conduction heat transfers in in-plane direction and through-thickness direction are treated independently. Since the significant portion of the conduction heat transfer in a PCB occurs in the in-plane direction in the conductor layers, this is a valid assumption. Under steady-state conditions and with constant thermophysical properties, the in-plane (i-p) conduction heat transfer equation for a PCB can be written as:

$$Q_{\text{in-plane}} = Q_{1\text{i-p}} + Q_{2\text{i-p}} + \ldots + Q_{\text{ni-p}} \tag{2-1}$$

where the subscript refers to the layers of the PCB. Using the conduction rate equation in rectangular coordinates for a PCB with a width of W, a length of L, layer thicknesses t_i and layer thermal conductivities k_i, Eq. (2-1) can be rewritten as:

$$W\,\Sigma t_i\,k_{in\text{-}plane}\,(T_{L=0} - T_{L=L})/L = W\,t_1\,k_1\,(T_{L=0} - T_{L=L})/L \\ + W\,t_2\,k_2\,(T_{L=0} - T_{L=L})/L \\ + W\,t_n\,k_n\,(T_{L=0} - T_{L=L})/L \qquad (2\text{-}2)$$

In-plane conduction heat transfer in a PCB represents a parallel thermal resistance circuit which can be written as:

$$(1/R_{in\text{-}plane}) = (1/R_1) + (1/R_2) + \ldots + (1/R_n) \text{ where } R_i = L/(k_i t_i W) \qquad (2\text{-}3)$$

where

$$k_{in\text{-}plane} = \Sigma(k_i t_i)/\Sigma t_i \qquad (2\text{-}4)$$

Through-thickness (t-t) conduction heat transfer in a PCB represents a series thermal resistance circuit, and the through-thickness conduction heat transfer equation for a PCB can be written as:

$$Q_{through\text{-}thickness} = Q_{1t\text{-}t} = Q_{2t\text{-}t} = \ldots = Q_{nt\text{-}t} \qquad (2\text{-}5)$$

which can be expanded into following equations:

$$W\,L\,k_{through\text{-}thickness}\,(T_{t=0} - T_{t=\Sigma ti})/\Sigma t_i = W\,L\,k_1\,(T_{t=0} - T_{t=t1})/t_1 \\ = W\,L\,k_2\,(T_{t=t1} - T_{t=t2})/t_2 = \ldots = W\,L\,k_n\,(T_{t=tn\text{-}1} - T_{t=tn})/t_n \qquad (2\text{-}6)$$

Inter-layer temperatures can be eliminated from Eqs. (2-6), and a series thermal resistance equation extracted as follows:

$$R_{through\text{-}thickness} = R_1 + R_2 + \ldots + R_n \text{ where } R_i = t_i/k_i \qquad (2\text{-}7)$$

where

$$k_{through\text{-}thickness} = \Sigma t_i/\Sigma(t_i/k_i). \qquad (2\text{-}8)$$

A printed circuit board is commonly built as layers of conductors separated by layers of insulators. The conductors are mostly alloys of copper, silver or gold, while the insulators are mostly a variety of