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Mathematical Models

in

Computer Vision

and Image Processing

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Vision is the art of seeing things invisible.
— Jonathan Swift
image: a reproduction or imitation of the form of a person or thing.
— Webster's (1968)
Vision is just a bag of tricks.
— Ramachandran, [Ram91]
Wir müssen wissen. Wir werden wissen.
We must know. We shall know.
— David Hilbert

Preface

In this course, we will study some mathematical models and problems associated with basic problems in computer vision and digital image processing. The mathematical models are set up with various mathematical theories, ranging from Bayesian inference approach, Markov random fields, variational calculus, scale space theory, partial differential equations, to stochastic differential equations.

A digital image is a matrix of positive integers which represents a pattern of radiant energy emitted by objects in apace. The image *synthesis* (or *direct*) problem is to determine the digital image given the scene geometry, sources of illumination, and other constraints, and is in the domain of optics, sensor modeling, and computer graphics. This course is concerned with image *analysis*, a general term encompassing problems in image processing, such as removing the effects of blur and noise, and those in computer vision, which involve geometric and semantical scene descriptions. It focus on the extraction of information implicit or hidden in the recorded digital image data by automatic devices aiming at an interpretation of the data. Whereas specific goals are application-dependent, one can still isolate many generic issues and sub-problems, all involved with converting information which is *implicit* or *hidden* in the recorded digital image to *explicit* descriptions of the physical world, a process sometimes characterized as "inverse optics" [Gem91], [PTK85].

Inverse problems generally are under-determined or ill-posed. Similarly various interpretations may be more or less compatible with the data and the art of image analysis is to select those of interest, or those most probable. Here is a selection of typical problems:

— Image denoising and enhancement.

— image restoration/reconstruction: recover a 'true' two- or three-dimensional scene from, usually noisy, data.

— Boundary detection: locate boundaries of individual objects inside an image.

— Image segmentation: partition the image into subregions with homogeneous intensity, color or texture, which will hopefully correspond to object or object parts. each partition uniquely defines a boundary map, whereas only boundary maps which are suitable organized will yield useful segmentation.

— Texture models and segmentation: partition images into regions of similar textures.

— Shape from shading or from other features: reconstruct a three-dimensional scene from the observed two-dimensional image.